Variance Estimation for Product Sales in the 2017 Economic Census: Utilizing Multiple Imputation to Account for Sampling and Imputation Variance

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**The views expressed in this presentation are those of the authors and not necessarily those of the U.S. Census Bureau*

Economic Census Background

- Not strictly a census
  - Multi-units and large single-units selected with certainty
  - Small single-units sampled

Economic Census Background
Data Items Collected

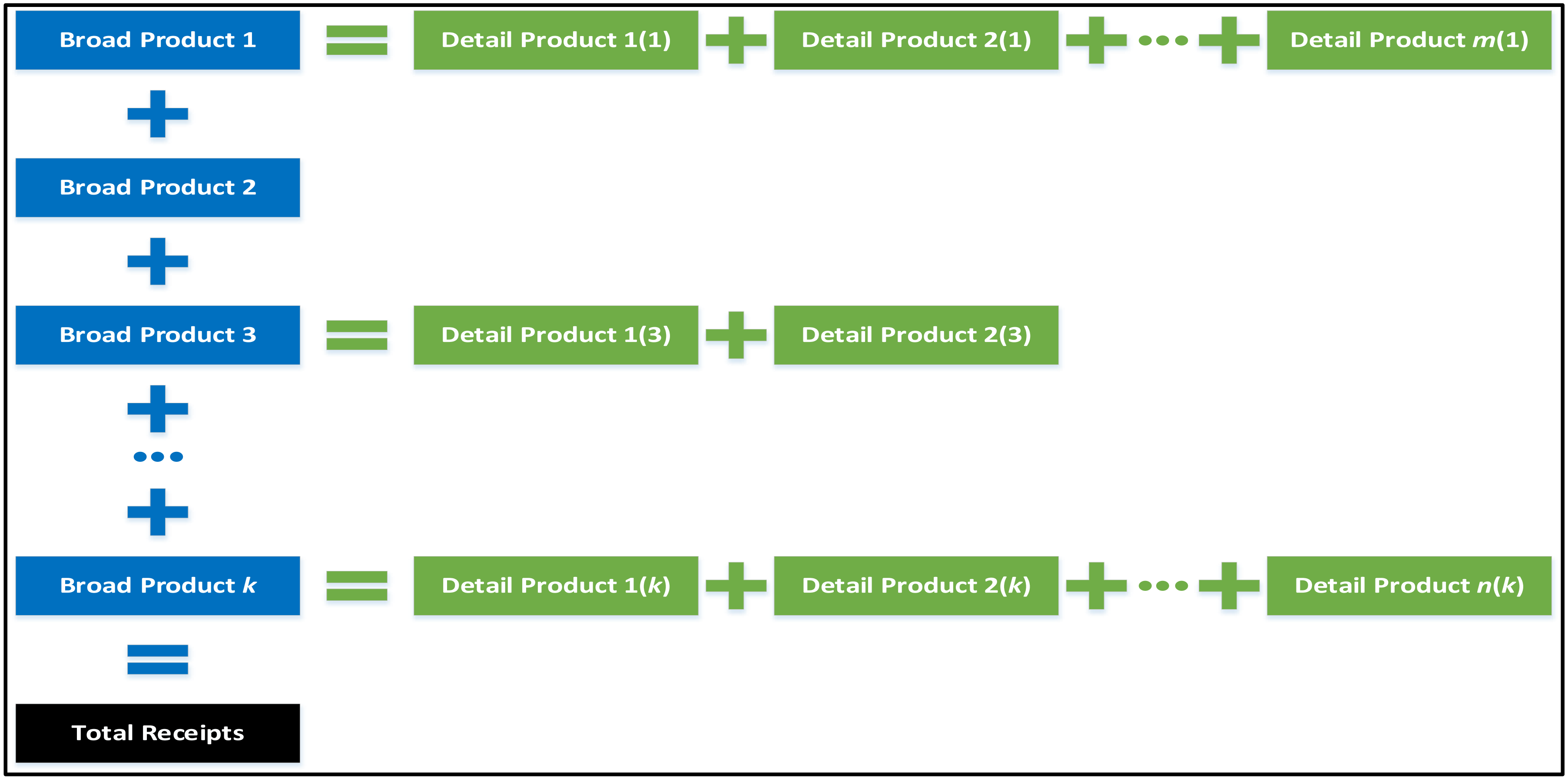
"General Statistics"

- Examples: Total receipts, Annual payroll, and 1st quarter employment
- Complete universe created using administrative records and imputation

Product Sales

- Only asked of sampled establishments
- Sample weights used to account for non-sampled establishments
- Two types: broad and detail
- Final product sales estimates are produced by calibration to stratum-level receipt totals

Economic Census Background
Product Sales Data



Research Challenges

- Dedicated Team
  - Short time frame (≈ 12-15 months)
  - Relative inexperience of team members with variance estimation
- Magnitude of the problem
  - ≈ 1,000 industries and ≈ 8,000 products
- Historical data limitations
  - Classification differences (to NAPCS)
  - Collection differences (to electronic)
  - Unit collection differences (from varied to $1,000)

Research Team

Research Team

- ≈ 1,000 industries
- ≈ 8,000 products
  - Broad products
  - Detail products
- Calibration Weighting (i.e., post-stratification)

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Research Team

- ≈ ~~1,000~~ 21 industries
- ≈ ~~8,000~~ Top 4 products
    - Broad products
    - ~~Detail products~~
- Calibration Weighting (i.e., post-stratification)

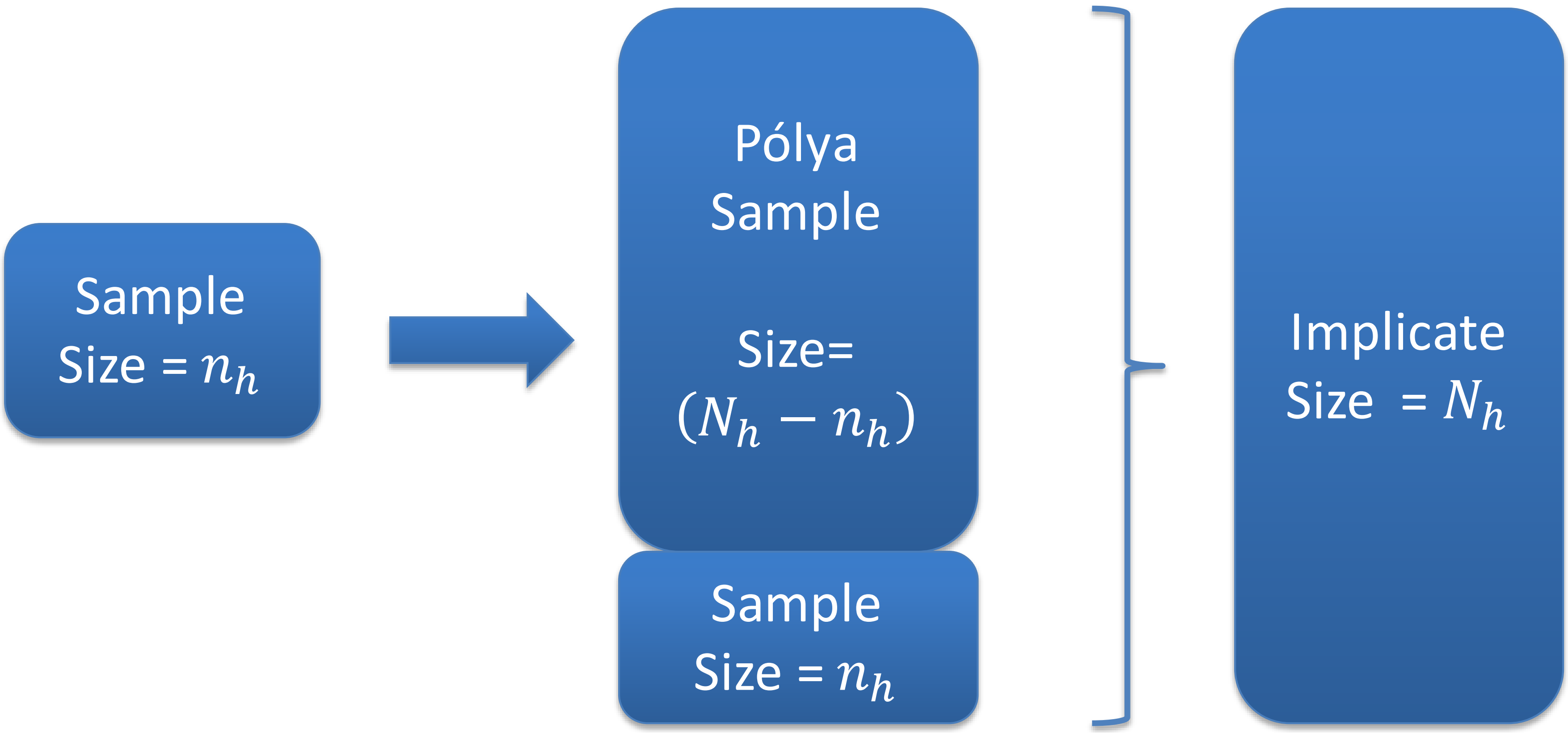
Research Evaluation

Perform simulation studies

- Two initial studies
  - Sampling Variance (Recommend: FPBB)
  - Variance Due to Imputation (Recommend: ABB)
- Final simulation of recommended method

- Recommendation: **FPBB-ABB**

Finite Population Bayesian Bootstrap (FPBB)



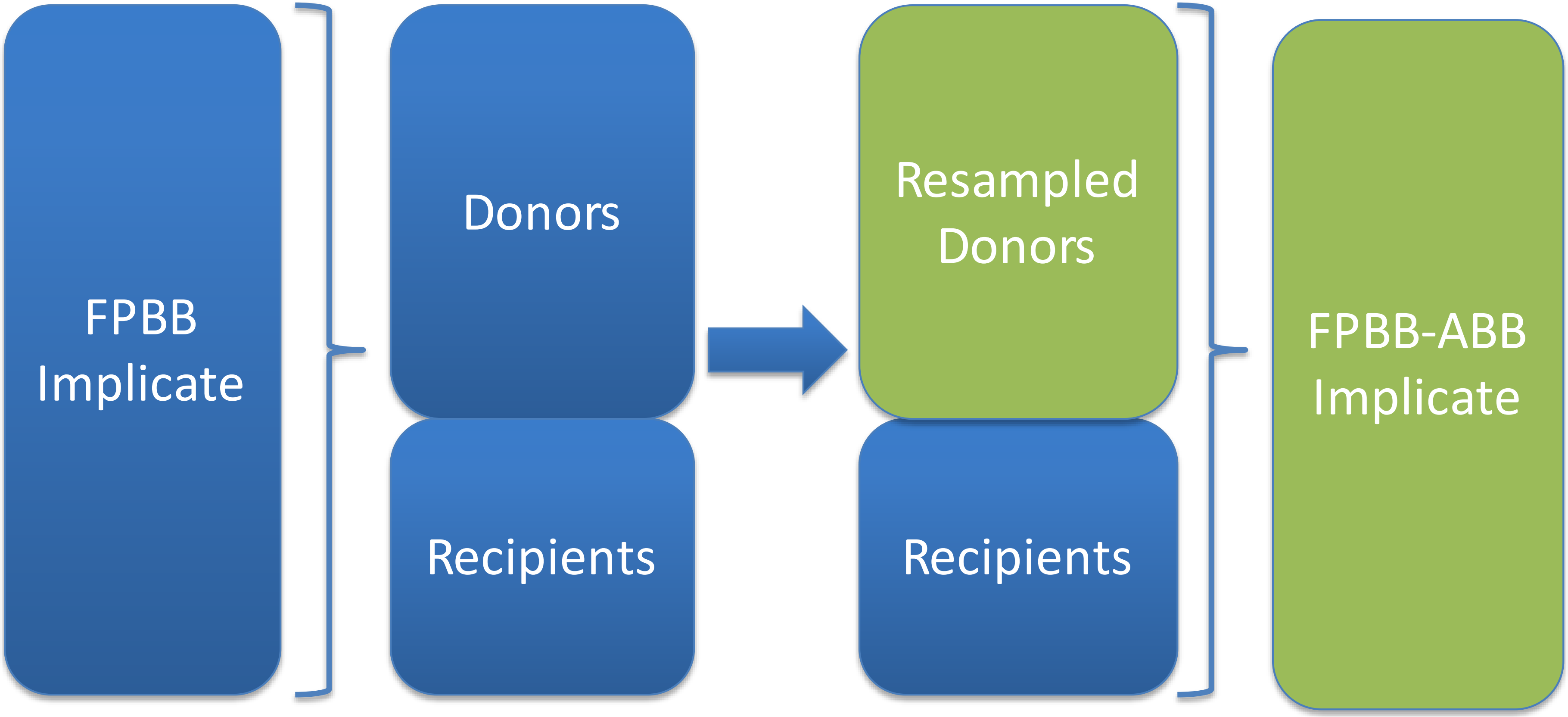
Finite Population Bayesian Bootstrap (FPBB)

- Create an implicate by drawing $N_h - n_h$ establishments from the sample with probability for the kth selection

$$p_{h,k} = \frac{\left(w_i - 1 + \frac{l_{i,k-1}(N_h - n_h)}{n_h}\right)}{N_h - n_h + \frac{(k_h - 1)(N_h - n_h)}{n_h}}$$

- Add the $N_h - n_h$ selected establishments to the original sample to complete the implicate

Approximate Bayesian Bootstrap (ABB)



FPBB-ABB

The FPBB-ABB estimate of variance is

$$\hat{V}_{final} = \hat{V}_{samp} + \frac{1}{B}\hat{V}_{imp}$$

- $\hat{V}_{samp} = \left(1 + \frac{1}{B}\right)\left(\frac{1}{B-1}\right)\sum_{b=1}^{B}[FPBBAVG_b - AVG]^2$
- $\hat{V}_{imp} = \left(1 + \frac{1}{C}\right)\left(\frac{1}{C-1}\right)\sum_{b=1}^{B}\sum_{c=1}^{C}\left[TOT_{b,c} - FPBBAVG_b\right]^2$

- B is the number of FPBB implicates
- C is the number of ABB implicates

Implementation Team

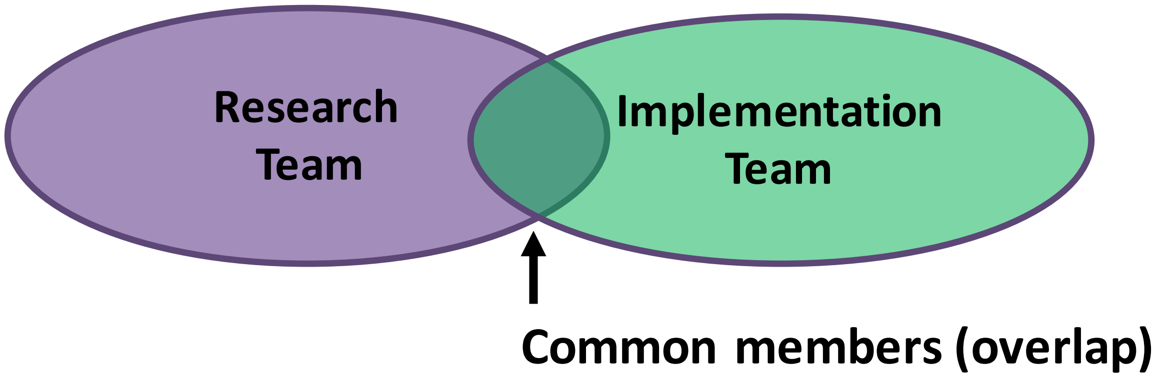
Research Team

- ~~1,000~~ 21 industries
- ~~8,000~~ Top 4 products
  - Broad products
  - ~~Detail products~~
- Calibration Weighting (i.e., post-stratification)

Implementation Team

- ≈ 1,000 industries
- ≈ 8,000 products
  - Broad products
  - Detail products
- Calibration Weighting (i.e., post-stratification)
- “Non-donors”
- Zero Receipts cases
- Processing time …

Implementation Team



- Overlap: Consultants
- New members
  - Subject Matter Experts
  - Programmers
  - Methodologists

Calibration Weighting

ID	Sample Weight	Calibration Factor	Adjusted Weight	$P_{h,k}$
1	15.000	0.48	7.200	1.240
2	4.000	0.48	1.920	0.184
3	2.000	0.48	0.960	-0.008
4	1.000	0.48	0.480	-0.104
5	1.000	0.48	0.480	-0.104
6	1.000	0.48	0.480	-0.104
7	1.000	0.48	0.480	-0.104
Total	25.000		12.000	

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Calibration Weighting

- Recall the formula for $p_{h,k}$

$$p_{h,k} = \frac{\left(w_i - 1 + \frac{l_{i,k-1}(N_h - n_h)}{n_h}\right)}{N_h - n_h + \frac{(k_h - 1)(N_h - n_h)}{n_h}}$$

- Use $w_i = Sample\ Weight$

Calibration Weighting

- Instead, introduce the calibration when calculating variance

- Further simplify by producing CVs

$$CV = \frac{\sqrt{Calibration^2 * Variance}}{Calibration * MI\ Estimate}$$

Calibration Weighting

- Instead, introduce the calibration when calculating variance

- Further simplify by producing CVs

$$CV = \frac{\sqrt{\cancel{Calibration^2} * Variance}}{\cancel{Calibration} * MI\ Estimate}$$

Proper ABB Implementation

ID	Sample Weight	Donor/Recipient
1	1.000	Donor
2	1.000	Donor
3	1.000	Donor
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Proper ABB Implementation

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$$Expected\ Variance = 0$$

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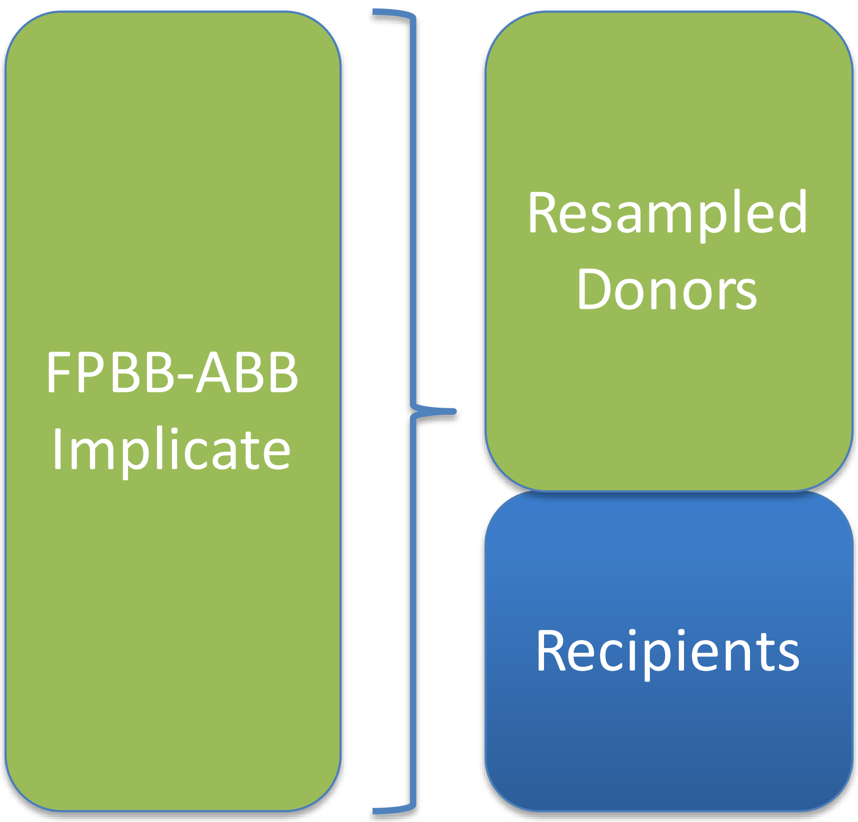
$$Expected\ Variance = 0$$

$$Variance\ Estimate \gg 0$$

Proper ABB Implementation



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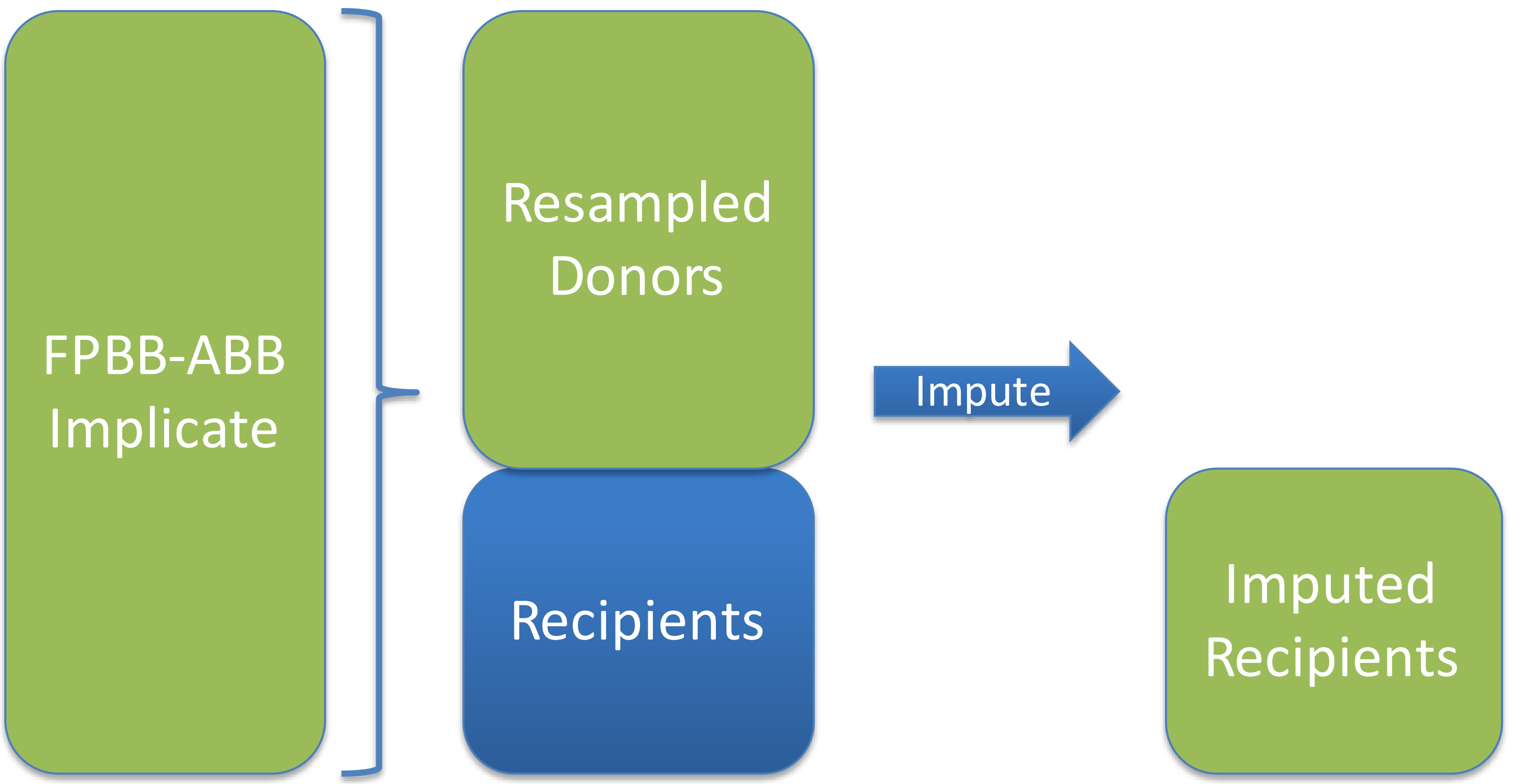

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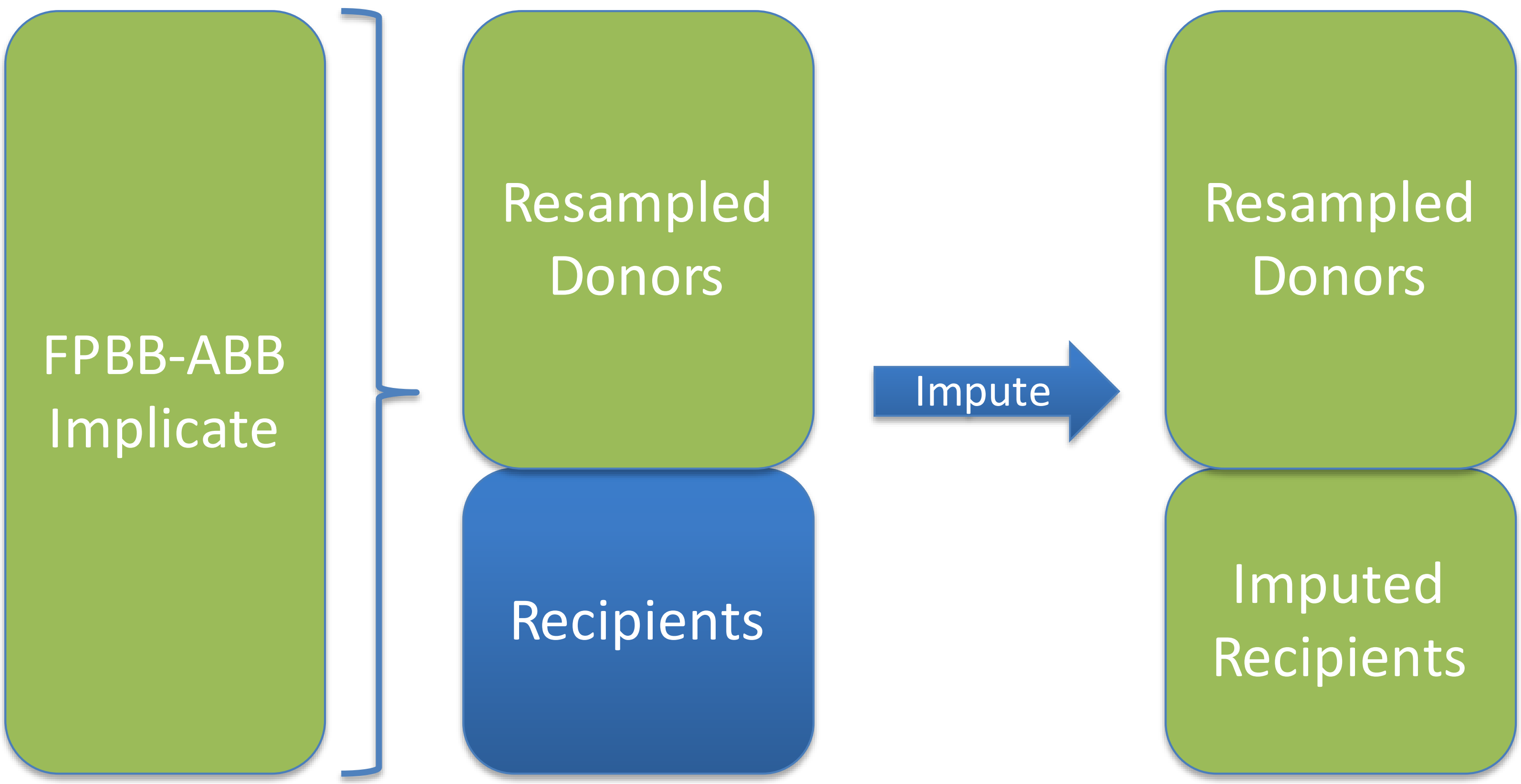

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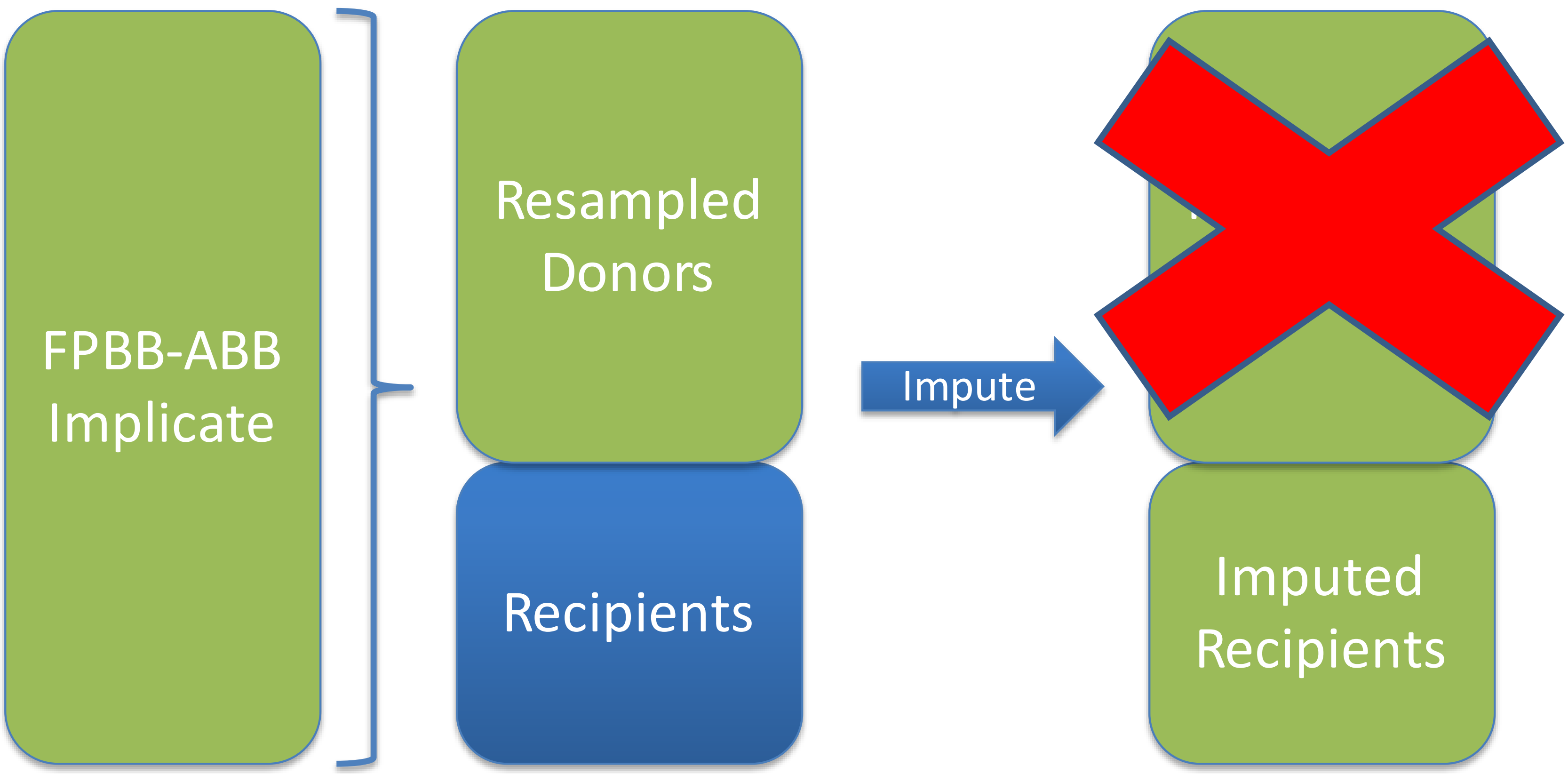

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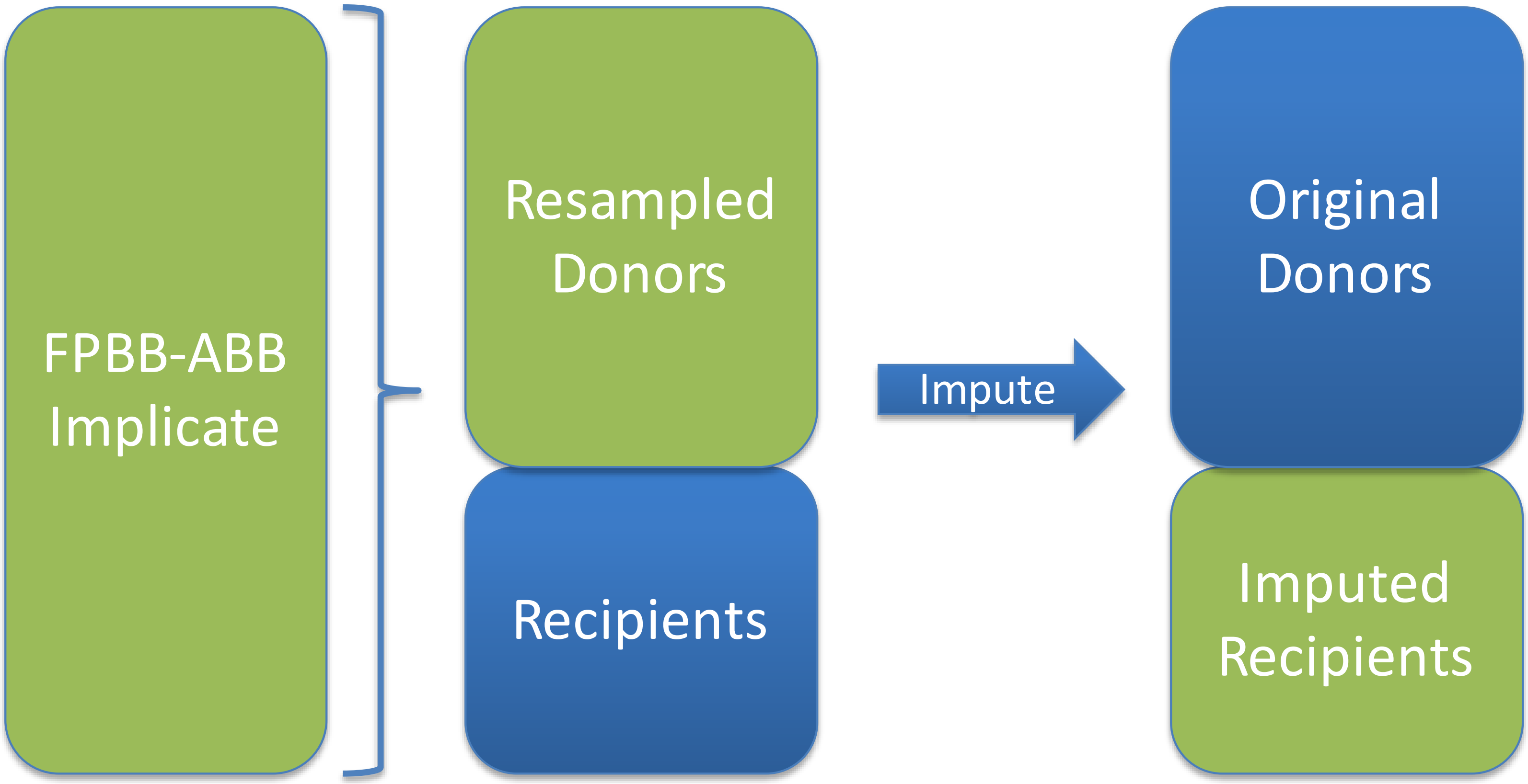

Proper ABB Implementation

Coverage Rates for 90% Confidence Intervals - Manufacturing

Response Rate	Product 1		Product 2	
	Resampled	Original	Resampled	Original
40	89.30%	76.70%	89.90%	78.50%
50	94.10%	77.10%	93.60%	76.60%
60	95.00%	72.50%	96.50%	73.80%
70	98.20%	69.50%	98.10%	69.40%
80	99.20%	65.70%	99.90%	69.70%
90	100.0%	61.70%	100.00%	65.10%

- Using resampled donors generally results in over coverage
  - Note, close to nominal coverage at lower response rates
- Using original set of donors results in under coverage

Proper ABB Implementation

Coverage Rated for 90% Confidence Intervals - Manufacturing

Response Rate	Product 1			Product 2		
	Resampled	Original	2xOriginal	Resampled	Original	2xOriginal
40	89.30%	76.70%	89.80%	89.90%	78.50%	90.80%
50	94.10%	77.10%	90.50%	93.60%	76.60%	90.00%
60	95.00%	72.50%	87.40%	96.50%	73.80%	88.90%
70	98.20%	69.50%	84.70%	98.10%	69.40%	83.80%
80	99.20%	65.70%	82.60%	99.90%	69.70%	84.90%
90	100.0%	61.70%	79.70%	100.00%	65.10%	81.80%

- Xie and Meng (2017) argues that hot deck imputation models are always uncongenial so that multiply imputed variance estimates are always underestimates
- Showed that the correct coverage is obtained by doubling the variances

Detail Products

Donors	Broad products usable
Complete	All broad and detail products usable
Partial	All broad products usable and some detail products usable
Minimal	All broad products usable; detail products missing and required
Recipients	Missing products
Full	Need broad and detail products
Partial	Need some (designated) detail products
Minimal	Need all detail products
Ineligible	All products usable, but not "typical"; excluded from donor pool

- Partial and minimal donors are "completed" using category average imputation
- Note, partial and minimal donors are:
  - Donors for imputation of broad products
  - Recipients for imputation of detail products

Detail Products

Options

1. Treat partial/minimal donors as donors for ABB
   - Correctly identifies broad products as donors
   - Incorrectly identifies detail products as donors
2. Treat partial/minimal donors as recipients for ABB
   - Correctly identifies detail products as recipients
   - Incorrectly identifies broad products as recipients
3. Run variance estimation separately for broad and detail products
   - Correctly identifies both broad and detail products
   - Very resource intensive (using unplanned resources)

Detail Products

Options

1. Treat partial/minimal donors as donors for ABB

- Leaves broad products unaffected
- "Complete" partial/minimal donors using category averages from each FPBB population

Conclusions

There was clear added value from the implementation team

- No real stand-out performer from initial research
- Expanding the scope for implementation revealed a lot of details not seen in the research phase

Acknowledgments

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