Metrics for Diversity, Equity, and Inclusivity: A Review

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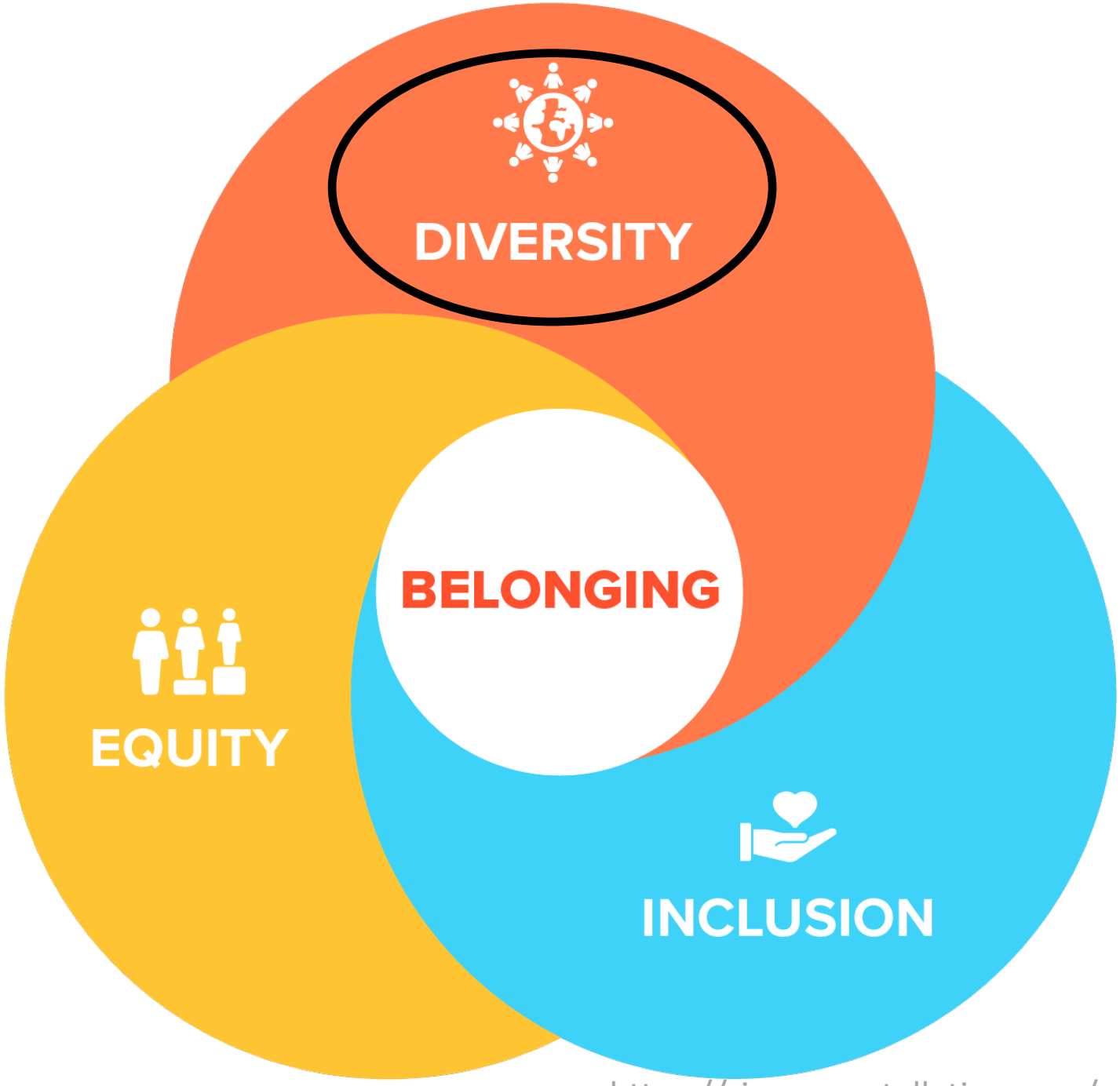

https://sierraconstellation.com/careers/diversity-equity-inclusion/

Metrics for Diversity (with discussion of Equity, and Inclusivity): A Review

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Primary Reference: Budescu, D.V., and Budescu, M. (2012) How to Measure Diversity When You Must, *Psychological Methods,* 17 (2): 215-227.

Diversity Measures: Notation and Setup

- Let $p_i =$ proportion of cases in Category $i, \quad i = 1, \ldots, C$
  - $p_i \geq 0 \quad \forall i$
  - $\sum_{i=1}^{C} p_i = 1$
- C "mutually exclusive and exhaustive" proportions ⇒ multinomial distribution

- Diversity measure, $D = f(p_1, p_2, \ldots, p_C)$ satisfies
  - D is bounded: $f(p_i = 0 \; \forall i) \leq D \leq f\left(p_i = \frac{1}{C} \; \forall i\right)$
  - D is invariant across all transformations that preserve the identity and integrity of C categories

Diversity Measure: Majority-Minority Approach

- Assumes $C = 2$
- Framework: let

  p_1 = proportion of individuals in C_1

  $p_2 = 1 - p_1 =$ proportion of individuals not in C_1
- Approach: let $D = p_1 p_2 = p_1(1 - p_1)$
- $D = 0.25$ maximum when $p_1 = p_2 = 0.5$
- $D \to 0$ when either p_1 or $p_2 \to 1$
- "Approach doesn't capture true diversity because limited to two groups"

Diversity Measure: Generalized Variance

$$GV = \sum_{i=1}^{C} p_i(1 - p_i) = 1 - \sum_{i=1}^{C} p_i^2$$

- Also referred to as Simpson's Index (Simpson, 1949)
- Properties:
  - invariant under categorical permutations and relabeling
  - $0 \leq GV \leq \frac{C-1}{C}$
- Normalized GV (NGV) allows for better comparison of distributions with differing number of categories:

$$NGV = \frac{GV}{\max(GV)} = \frac{C}{C-1}\left(1 - \sum_{i=1}^{C} p_i^2\right)$$

- $NGV = -ES(\chi^2)$

Hypothesis Test for Uniformity

- $H_0: p_i = \frac{1}{C} \; \forall i = 1, \ldots, C$
- Test statistic:

$$\chi^2 = \frac{\sum_{i=1}^{C} \left(p_i - \frac{1}{C} \right)^2}{1/C} = C \sum_{i=1}^{C} p_i^2 - 1$$

- $\frac{\chi^2}{C-1} = 1 - NGV$

Diversity Measure: Entropy

$$H = -\sum_{i=1}^{C} p_i \log_2(p_i)$$

- $0 \leq H \leq \log_2(C)$
- Shannon's diversity index (i.e. Shannon-Weiner) defines H via ln vs. $\log_2$
- Like GV, sensitive to number of categories/groups, so alternative is normalized entropy:

$$NH = \frac{H}{\max(H)} = -\sum_{i=1}^{C} p_i \log_2(p_i) / \log_2(C)$$

- NH also referred to as an evenness index (Pielou 1966)

LRT for Uniformity

- $H_0: p_i = \frac{1}{C}\ \forall i = 1, \ldots, C$

- Test statistic:

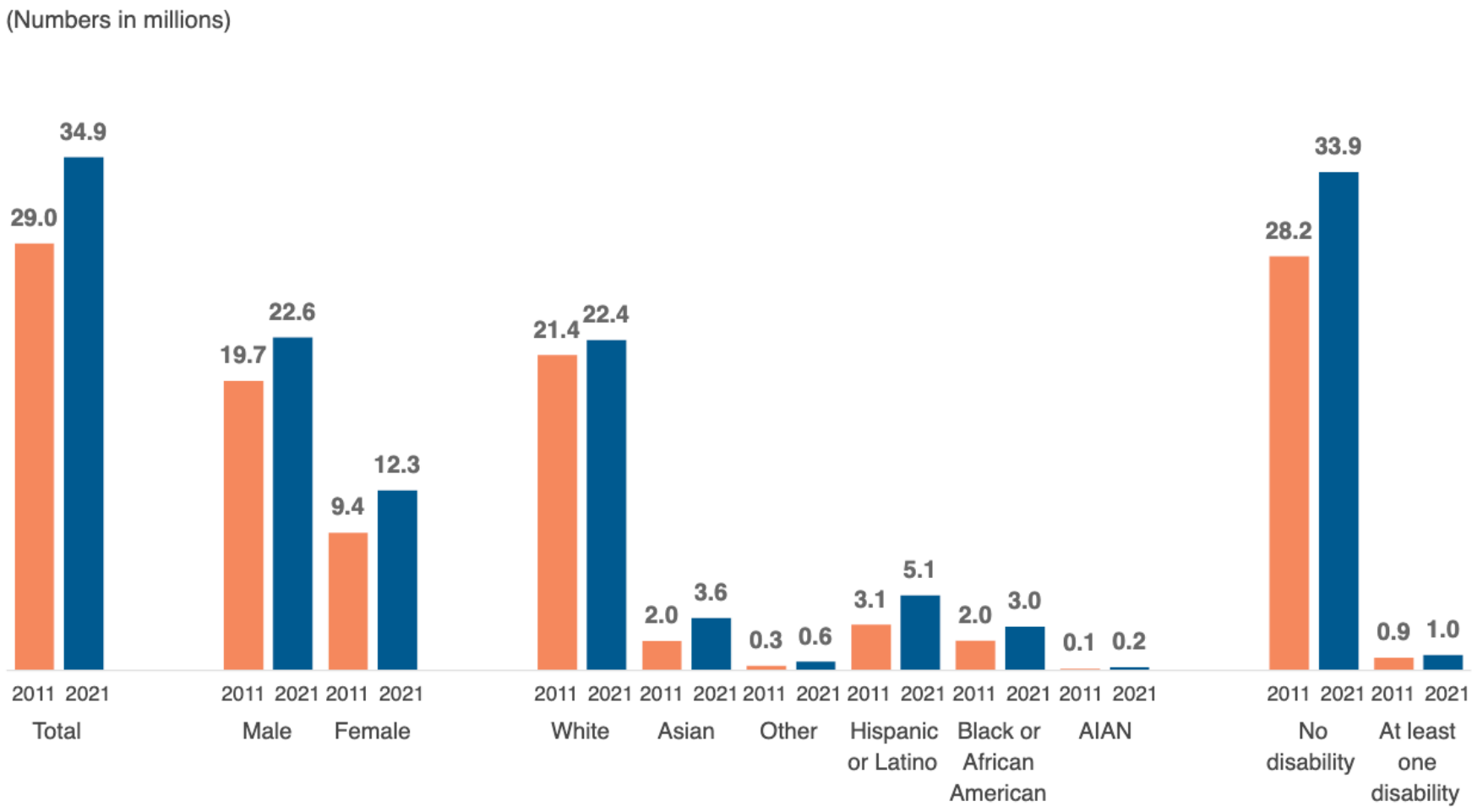
$$X^2 = -2\sum_{i=1}^{C} p_i \ln\left(\frac{p_i}{1/C}\right) = 2.886[\log_2(C) - H] = 2.886(1 - NH)$$

Example:

Growth in the S

**The size of the STI
all groups.**

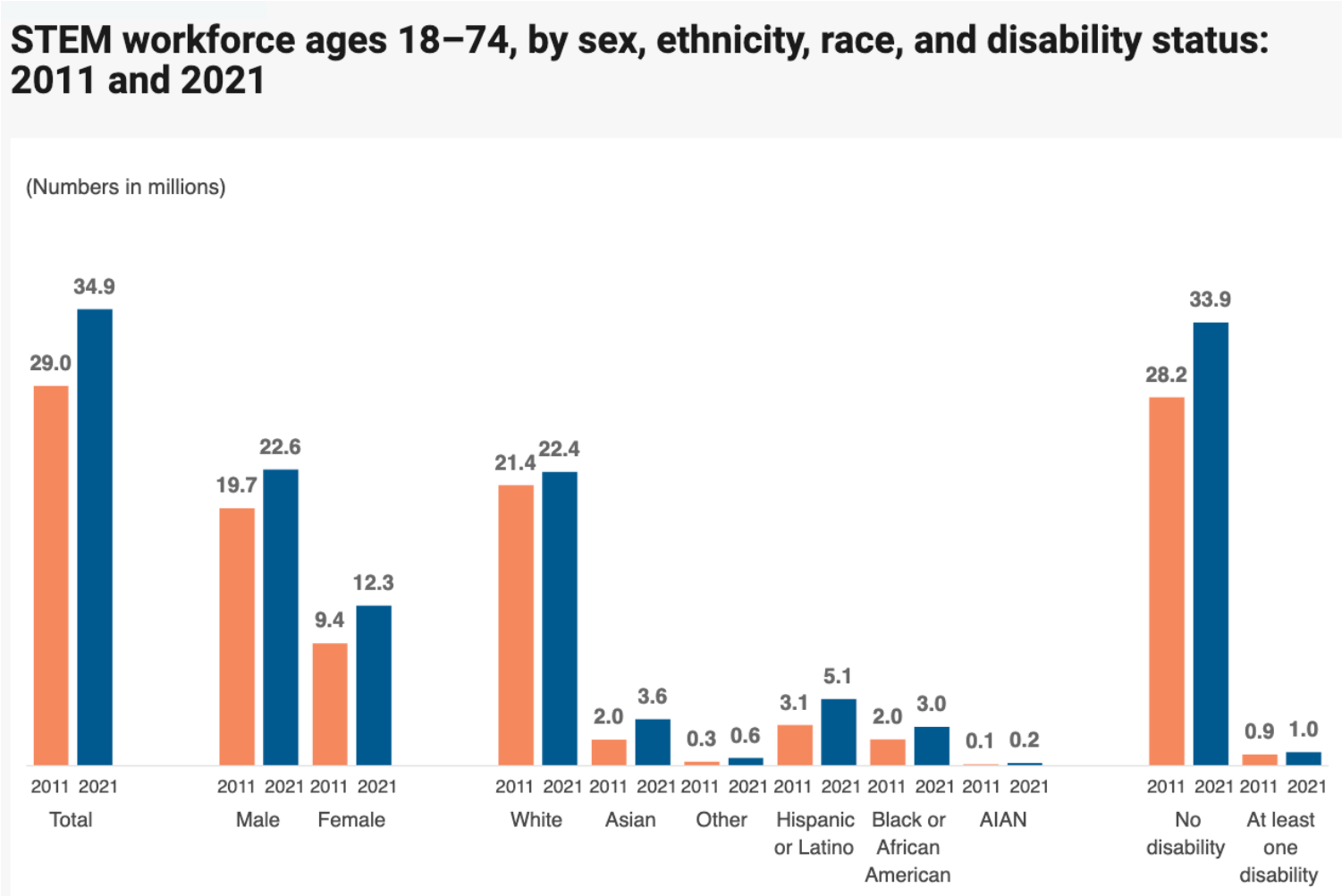
STEM workforce ages 18–74, by sex, ethnicity, race, and disability status: 2011 and 2021



Question: Is there an improvement with regard to diversity?

Answer:

	Diversity Index	2011	2021
Sex	GV	0.4335	0.4564
	H	0.4282	0.4498
Race and Ethnicity	GV	0.4344	0.5483
	H	0.3131	0.3858
Disability Status	GV	0.0534	0.0557
	H	0.0430	0.0438



- *GV* measures are slightly higher than *H*
- High correlation between *GV* and *H*

Summary

- Indexes provide single numerical value describing the amount of (dis)similarity between the relative size of C subpopulations defined by demographic and social categories
- Measure of scatter for categorical variables
- The “majority-minority” approach is insufficient
- “GV generalizes the binomial distribution variance, easily interpreted as the likelihood of randomly picking out two individuals from the different groups in the population, and directly relates to Pearson chi-square test of uniformity and its associated measure of effect size”
- Entropy (H) is based on information theory and related to the chi-squared test of uniformity and its associated measure of effect size
- GV, H very highly correlated

Summary (cont.): Special cases

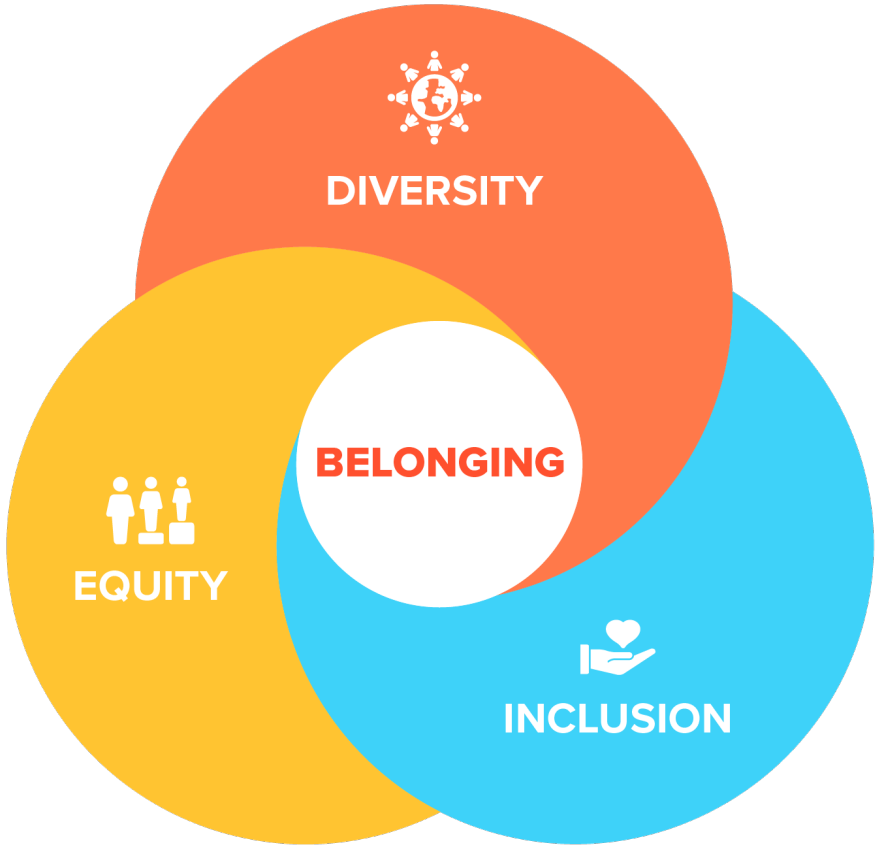
- Binomial distributions ($C = 2$)
  - NGV and NH become closer to each other as (a) the two proportions are more similar to each other (i.e. $NGV = NH = 1$), and (b) when population is concentrated in one of the groups ($NGV = NH = 0$)
  - NGV, NH farthest apart when one group comprises 90% of the population and the other makes up the remaining 10%
- Trinomial distributions ($C = 3$)
  - NGV, NH most divergent when population equally divided between two of the groups and the third group is empty

Summary: How many categories?

- Distribution of GV and H statistics affected when 2+ categories combined to form one, and data recoding affects explanatory power of diversity
- Determine whether the extra categories increase the differentiation between the various cases.

Discussion

- Precise vs imprecise classification
- Appropriate test for diversity?
- Indexes for inclusivity and equity



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< Inclusiveness Index

Inclusiveness Index

Overview

The Othering and Belonging Institute's "Inclusiveness Index" is a holistic gauge of the degree of inclusivity experienced by marginalized groups across the globe and within the United States. Our index ranks states and countries in absolute and relative terms using a variety of indicators. Our instrument is unique in striving to gauge inclusivity on its own terms rather than as part of a more general assessment of group well-being, wealth or economic conditions.

Inclusiveness Index: Methodology

- Data for indicators collected and processed for analysis as z-scores
  - “Z-score calculated for all indicators in each dimension – Race, Gender, LGBTQ+, Religion, Disability and General Population, and adjusted (multiplied by ‘-1’) where higher values of indicators meant lack of inclusion (e.g. higher index values for government restrictions on religion)”
- Z-score values then scaled from 0-100 for each indicator
- Dimension index score = average of scaled scores of each indicator within the dimension
- Inclusiveness Index value = average of index scores for all dimensions
  - Inclusiveness level (high to low) determined by sorting data in descending order, categorizing into quintiles

National Science Foundation
Directorate for Social, Behavioral and Economic Sciences

Analytics for Equity Initiative phase 1 solicitations posted

Analytics for Equity proposals due March 3, 2023

1. Equity of access to STEM Research and Education Opportunities **(Agency Partner: NSF)**
2. Environmental Stressors and Equity **(Agency Partner: EPA)**
3. Equity in service delivery and supports including childcare, food security, or economic support **(Agency Partner: HHS ASPE)**
4. Health Equity in the Wake of Climate Change **(Agency Partner: HHS CDC)**
5. Equity considerations for Workplace Safety and Workers **(Agency Partner: DOL)**

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